MATH CHALLENGERS®

Friday, January 29th, 2016
★ Mock Competition ★
Co-op Round
Problems 1 – 15

School/Team Code

Grade(s)

Team Members

Team Captain

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

Number of Problems: 15
Time Allotted: 36 minutes
Scientific calculators are permitted, but books or other aids are *not* permitted

Answer in exact form and round only when asked to do so. No units need to be provided after your answers. Please record only final answers in the blanks in the left-hand column of the competition paper. If you complete the problems before time is called, use the remaining time to check your answers.

Only **one** copy of your team's Co-op Round competition paper will be collected for marking. *If you wish for this paper to be collected for marking, please check the box below:*

☐

FORM CODE

A		F		0		5	
B	■	G		1	■	6	
C		H		2		7	
D		I		3		8	
E		J		4		9	

Total Correct	Scorer's Initials

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1. $ _____ **1.** Alphonse has twice the amount of money as Beryl. Between them, they have $216 in total. How much more money does Alphonse have than Beryl, in dollars?

2. _____ **2.** Two gears are attached to each other. Gear A has 72 teeth and gear B has 88 teeth. Right now one of the teeth from each gear are touching each other. What is the minimum number of rotations that Gear A have to turn in order for the two teeth to touch each other again?

3. _____ **3.** Define $\pi(n)$ as the number of positive divisors of the number n. How many integers n with $1 \leq n \leq 100$ satisfy the property that $\pi(n) = 3$?

4. _____ **4.** There are two right angle triangles, ΔABC and ΔDEF. ΔABC has one leg measuring 3. ΔDEF has one leg measuring 5, one leg measuring 3 longer than the hypotenuse of ΔABC , and a hypotenuse 5 longer than ΔABC's unknown leg. What is the length of ΔABC's hypotenuse?

5. _____ **5.** A math contest with 5 questions has the following rules for scoring:

1. A full, correct solution is awarded 2 points
2. A partial solution is awarded 1 points
3. A wrong or blank solution is awarded 0 points

No other scores are allowed. What is the minimum number of people who must take the contest in order to guarantee that at least three students will get the same score?

6. _____ **6.** A fraction is a defined as a ratio of two integers (i.e. $\frac{m}{n}$ for two integers m and n). How many fractions $\frac{m}{n}$ exist such that:

- $\frac{m}{n} < 2$
- m and n are both prime numbers
- $n < 20$

7. _____ **7.** Anna, Margaret, and 13 other people are divided into two groups, a group of 5 and a group of 10. What is the probability that Anna and Margaret end up in the same group?

8. _____ **8.** In square $ABCD$, diagonal AC has length 10. The angle bisectors of $\angle BAC$ and $\angle DAC$ intersect BC and CD at points E and F, respectively. EF intersects AC at point G. What's the length of AG? Express your answer in simplest radical form.

9. _____ **9.** A group of engineers and entomologists had a party. At the party, everybody ate either one hot dog or one celery stick, but not more. $\frac{5}{9}$ of the hot dogs were eaten by engineers, and $\frac{11}{20}$ of the celery sticks were eaten by entomologists. If an equal number of engineers and entomologists attended the party, what fraction of all party-goers ate hot dogs?

10. _____ **10.** Express $\sqrt{2+\sqrt{3}}$ in the form $\frac{\sqrt{a}+\sqrt{b}}{c}$ where a, b, c are integers, and neither a nor b is divisible by the square of any prime.

11. _____ **11.** Evaluate $\frac{x^2-y^2}{x+y}$ if $x^6 - 3x^4y^2 + 3x^2y^4 - y^6 = 250047$, where x and y are both positive integers smaller than 9.

12. _____ **12.** How many ways are there to pick 4 integers from 1 to 10 such that no two of the 4 integers have the same remainder when divided by 4?

13. _____ **13.** A square with side length $2\sqrt{65}$ lies in the coordinate plane. The square has one vertex at point $(4, 7)$, one vertex in quadrant III, and two vertices in quadrant IV. Two sides of the square have slope $-\frac{4}{7}$. The other three vertices of the square have coordinates $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3). What is the smallest value among y_1, y_2, and y_3?

14. _____ **14.** A geometric sequence of integers has the following properties:

1. The common ratio is equal to the square of a positive integer and is larger than 1
2. The 5th term of the sequence is 85293

What is the square root of the product of the 29th term and the 1st term? Express your answer in the form of $a^b \times c^d$, where $a > c$, a and c are both prime, and a, b, c, d are positive integers.

15. _____ **15.** An isosceles trapezoid has one vertex at $(0, 0)$ on the coordinate plane, with a distance of 5 between its two parallel sides, which are themselves parallel to the x axis. The area of the trapezoid is equal to the sum of the positive divisors of 5!. One of the two legs of the trapezoid lie on a line with slope $\frac{5}{3}$. The other three vertices of the trapezoid have coordinates $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3). If the maximum value among $x_1 + y_1, x_2 + y_2$, and $x_3 + y_3$ is M, what's the maximum possible value for M?